

Chapter 5 — The Performance of Feedback Control Systems

Dorf & Bishop, Modern Control Systems (14th ed.)

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5.1 Introduction: Why performance matters

- Feedback lets us trade off **transient** vs. **steady-state** performance.
- Performance is specified via time-domain indices (speed, overshoot, settling) and steady-state accuracy.
- Design is about *compromise*: minimizing one metric can worsen another.

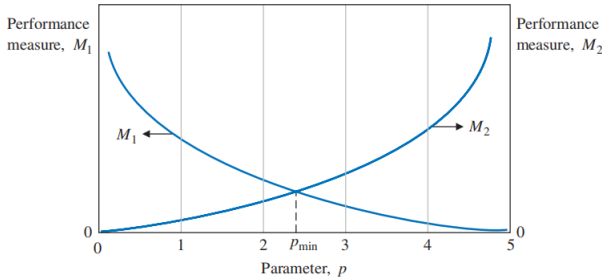
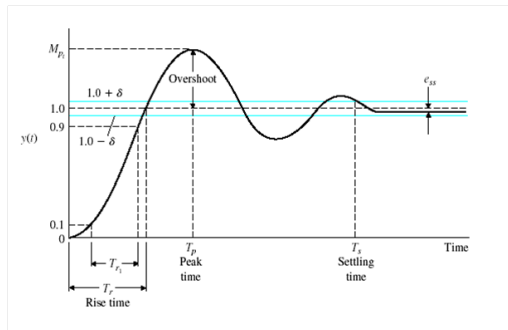


Figure: Design compromise curve

“Two performance measures vs parameter p ”

Performance measures at a glance — Time-domain picture

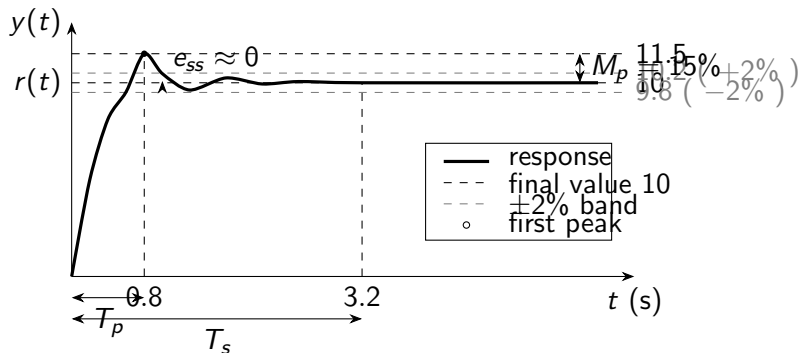


- **Speed:** rise time T_r , peak time T_p
- **Shape:** overshoot %OS, settling time T_s
- **Accuracy:** steady-state error e_{ss}

Example: Read Specs from Numbers

Given. Final value = 10. First peak = 11.5 at 0.8 s; within $\pm 2\%$ band by 3.2 s.

Compute. $M_p = \frac{11.5 - 10}{10} = 15\%$; $T_p = 0.8$ s; $T_s = 3.2$ s; $e_{ss} \approx 0$.



Checkpoint 5.1 (Poll)

- ① Which spec tells you how long until the response stays near the final value?
- ② If the first peak is 12 for a final value of 10, what is M_p ?
- ③ If the plot never crosses above the final value, what can you say about M_p ?

Checkpoint 5.1 — Detailed Solutions

- ① Settling time T_s (typically to a $\pm 2\%$ band).
- ② $M_p = \frac{12-10}{10} \times 100\% = 20\%$.
- ③ $M_p = 0\%$ (no overshoot).

Why standard test inputs?

- Real commands are varied/unknown; **standard inputs** allow comparison across designs and correlate with real performance. (step, ramp, parabolic, impulse)
- **We assume stability**; then the response to a test input exposes transient/steady-state performance.
- Common set: **step, ramp, parabolic**; also **impulse** via a limiting rectangular pulse.

Definitions and Laplace transforms (Table 5.1)

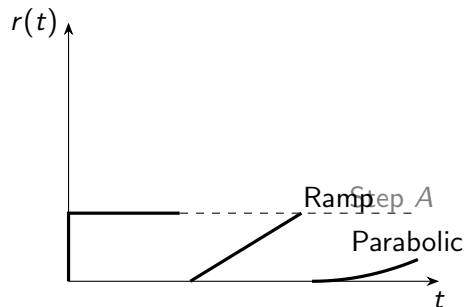
Signals (amplitude A): (see Table 5.1)

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$$\text{Step: } r(t) = A u(t) \Rightarrow R(s) = \frac{A}{s}$$

$$\text{Ramp: } r(t) = A t u(t) \Rightarrow R(s) = \frac{A}{s^2}$$

$$\text{Parabolic: } r(t) = \frac{A}{2} t^2 u(t) \Rightarrow R(s) = \frac{A}{s^3}$$



Integral relation: ramp = $\mathcal{I}\{\text{step}\}$, parabola = $\mathcal{I}\{\text{ramp}\}$. :contentReference[oaicite:7]index=7

General form and transform (linking all test inputs)

Monomials generate the canonical tests

For $n = 0, 1, 2, \dots$,

$$r(t) = t^n u(t) \implies R(s) = \mathcal{L}\{t^n u(t)\} = \frac{n!}{s^{n+1}}.$$

Thus, **step** ($n = 0$), **ramp** ($n = 1$), **parabola** ($n = 2$) are special cases.

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Impulse signal via limiting rectangle (definition)

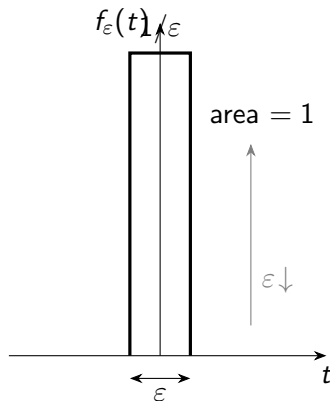
Start with a rectangle of width ε and area 1:

$$f_{\varepsilon}(t) = \begin{cases} \frac{1}{\varepsilon}, & |t| \leq \frac{\varepsilon}{2} \\ 0, & \text{otherwise} \end{cases} \rightarrow \delta(t) \quad (\varepsilon \rightarrow 0^+).$$

Properties (sifting/area):

$$\int_{-\infty}^{\infty} \delta(t) dt = 1, \quad \int_{-\infty}^{\infty} \delta(t - a) g(t) dt = g(a).$$

These motivate the **impulse test** and connect to the impulse response.



Convolution and the special role of the impulse

For a stable LTI system with impulse response $g(t)$ and input $r(t)$,

$$y(t) = \int_{-\infty}^{\infty} g(t - \tau) r(\tau) d\tau \iff Y(s) = G(s)R(s).$$

With $r(t) = \delta(t)$,

$$y(t) = \int g(t - \tau) \delta(\tau) d\tau = g(t),$$

so the **impulse response equals the output to a unit impulse.**

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Step-by-step: compute transforms (worked micro-examples)

Step: $r(t) = A u(t)$

$$R(s) = \int_0^{\infty} A e^{-st} dt = \left[-\frac{A}{s} e^{-st} \right]_0^{\infty} = \frac{A}{s}.$$

Ramp: $r(t) = A t u(t)$

$$R(s) = \int_0^{\infty} A t e^{-st} dt = A \cdot \frac{1}{s^2}.$$

Parabolic: $r(t) = \frac{A}{2} t^2 u(t)$

$$R(s) = \int_0^{\infty} \frac{A}{2} t^2 e^{-st} dt = \frac{A}{s^3}.$$

Impulse: $r(t) = \delta(t)$

$$R(s) = \int_{-\infty}^{\infty} \delta(t) e^{-st} dt = 1.$$

Mini example: output to a first-order system

Plant $G(s) = \frac{K}{\tau s + 1}$. Input **step** $R(s) = \frac{A}{s}$:

$$Y(s) = \frac{K}{\tau s + 1} \cdot \frac{A}{s} \Rightarrow y(t) = KA \left(1 - e^{-t/\tau} \right) u(t).$$

Ramp $R(s) = \frac{A}{s^2}$:

$$Y(s) = \frac{KA}{(\tau s + 1)s^2} \Rightarrow y(t) = KA \left(t - \tau + \tau e^{-t/\tau} \right) u(t).$$

(Compare *steady-state error* across step/ramp once we reach Sec. 5.6.)

Mini example: impulse response and convolution

For the same $G(s) = \frac{K}{\tau s + 1}$ and $r(t) = \delta(t)$,

$$Y(s) = G(s) \Rightarrow y(t) = g(t) = \frac{K}{\tau} e^{-t/\tau} u(t).$$

For a **general** $r(t)$: $y(t) = (g * r)(t) = \int_0^t g(t - \xi) r(\xi) d\xi$. This is the computational backbone behind all test inputs.

Where we use each test — and what “Type 1 / Type 2” mean

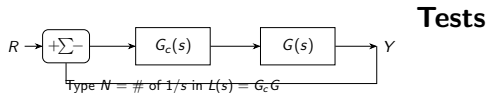
System type N (unity feedback): number of *pure integrators* in the open-loop

$$L(s) = G_c(s)G(s) = \frac{(\text{zeros})}{s^N (\text{other poles})}.$$

- **Type 0:** $N = 0$ (no integrator)
- **Type 1:** $N = 1$ (one $1/s$ factor)
- **Type 2:** $N = 2$ (two $1/s$ factors)

Steady-state error (SSE) to standard inputs:

Type	Step (A)	Ramp (At)	Parabolic ($\frac{A}{2}t^2$)
0	$\frac{A}{1 + K_p}$	∞	∞
1	0	A	∞



and what they probe

- **Step:** baseline transient & SSE (e_{ss})
- **Ramp:** tracking requires **type** ≥ 1 (K_v finite)
- **Parabolic:** tracking requires **type** ≥ 2 (K_a finite)

Concrete examples of Type 1 and Type 2

Type 1 example (one integrator): $G_c(s) = K$, $G(s) = \frac{b}{s(s+a)}$

$$L(s) = \frac{Kb}{s(s+a)} = \frac{Kb}{a} \cdot \frac{1}{s} \cdot \frac{1}{\left(1 + \frac{s}{a}\right)} \Rightarrow N = 1.$$

SSE: step $\rightarrow 0$; ramp $\rightarrow \frac{A}{K_v}$ with $K_v = \lim_{s \rightarrow 0} sL(s) = \frac{Kb}{a}$.

Type 2 example (two integrators): $G_c(s) = K$, $G(s) = \frac{b}{s^2(s+a)}$

$$L(s) = \frac{Kb}{s^2(s+a)} \Rightarrow N = 2.$$

SSE: step $\rightarrow 0$; ramp $\rightarrow 0$; parabola $\rightarrow \frac{A}{K_a}$ with $K_a = \lim_{s \rightarrow 0} s^2 L(s) = \frac{Kb}{a}$.

How to raise the type in practice:

- Add an **integrator** in the controller (e.g., PI) $\Rightarrow$ Type 0 $\rightarrow$ 1.
- Add another integrator in the loop (plant already has one) $\Rightarrow$ Type 1 $\rightarrow$ 2.

Poll: quick checks

Q1 What is $R(s)$ for $r(t) = 3t u(t)$?

Q2 If $r(t) = \frac{1}{2}t^2 u(t)$, what is $R(s)$?

Q3 For $G(s) = \frac{5}{s+5}$ and unit step input, what is $y(\infty)$?

Q4 For the same $G(s)$ and unit impulse input, what is $y(t)$?

Q5 Which input best probes *tracking ability* of type-1 systems?

A1 $R(s) = \frac{3}{s^2}$. **A2** $R(s) = \frac{1}{s^3}$.

A3 $y(\infty) = \lim_{t \rightarrow \infty} 5(1 - e^{-5t}) = 5$ (DC gain = 1).

A4 $y(t) = g(t) = 5e^{-5t}u(t)$.

A5 The **ramp** input (relates to K_v and ramp error).

Canonical closed-loop and setup

Closed-loop form (single-loop, unity feedback):

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

With unit step input $R(s) = 1/s$,

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}.$$

This is the standard 2nd-order template for relating pole locations to time-domain behavior.

Underdamped step response (form)

For $0 < \zeta < 1$, define $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ and $\beta = \sqrt{1 - \zeta^2}$. A standard time response form is

$$y(t) = 1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \theta), \quad \cos \theta = \zeta.$$

(Equivalent notations: the text parameterizes the same response via β and θ .)

Impulse response (link to step)

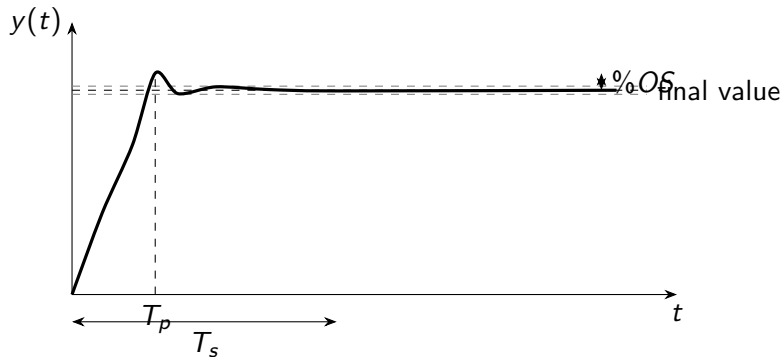
With $R(s) = 1$,

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \implies y(t) = \frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t).$$

The impulse response is the *derivative* of the step response.

Transient measures (definitions on the step)

- **Rise time** T_r : 0–100% (underdamped); use 10–90% if overdamped.
- **Peak time** T_p : time to first peak.
- **Percent overshoot** $\%OS = 100(M_p - 1)$.
- **Settling time** T_s : time to enter and stay within $\pm 2\%$ band.



Deriving T_p (peak time)

Start with $y(t)$ and set $\dot{y}(t) = 0$ for the first peak:

$$y(t) = 1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \theta).$$

$$\dot{y}(t) = 0 \quad \Rightarrow \quad \omega_d t + \theta = \pi \quad \Rightarrow \quad \boxed{T_p = \frac{\pi - \theta}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}}$$

(using $\cos \theta = \zeta$).

Deriving M_p and %OS

Evaluate $y(T_p)$:

$$M_p = y(T_p) = 1 + \exp\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right).$$

Hence

$$\boxed{\%OS = 100 e^{-\zeta\pi/\sqrt{1-\zeta^2}}}$$

These relations expose the ζ – overshoot trade-off.

Settling time rule-of-thumb

Dominant exponential envelope $\sim e^{-\zeta\omega_n t}$. For the $\pm 2\%$ band:

$$e^{-\zeta\omega_n T_s} \approx 0.02 \quad \Rightarrow \quad \boxed{T_s \approx \frac{4}{\zeta\omega_n}}.$$

This is a practical design heuristic to connect pole location to time-to-settle.

Big picture: why poles & zeros matter

- **Poles** (roots of the denominator) determine the *natural modes*: exponentials and damped sinusoids.
- **Zeros** (roots of the numerator) shape how those modes appear in the output (overshoot/undershoot, initial direction, “inverse response”).
- Strategy: read the s-plane (poles/zeros) $\Rightarrow$ predict key time metrics (T_p , %OS, T_s) and qualitative shape.

From transfer function to time response (step-by-step)

Closed-loop (or simple open-loop) transfer:

$$T(s) = \frac{Y(s)}{R(s)} = \frac{N(s)}{D(s)} = \frac{k \prod_{i=1}^m (s - z_i)}{\prod_{j=1}^n (s - p_j)}.$$

Partial fractions (no repeats):

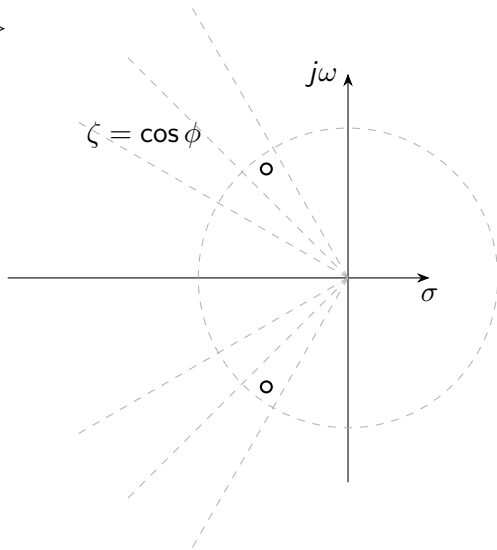
$$Y(s) = \sum_{j=1}^{n_r} \frac{A_j}{s - \sigma_j} + \sum_{k=1}^{n_c} \frac{B_k s + C_k}{(s - \alpha_k)^2 + \omega_k^2},$$

$$y(t) = \sum_j A_j e^{\sigma_j t} + \sum_k D_k e^{\alpha_k t} \sin(\omega_k t + \theta_k).$$

- Real pole $s = \sigma < 0 \Rightarrow e^{\sigma t}$ (pure decay).
- Complex pair $s = \alpha \pm j\omega \Rightarrow$ damped sinusoid $e^{\alpha t} \sin(\omega t + \theta)$.
- **Zeros** affect the *residues* (A_j, B_k, C_k) $\Rightarrow$ change shape (peaking/undershoot).

s-plane geometry $\Rightarrow$ transient feel

- **Decay speed:** distance to $j\omega$ axis (more left $\Rightarrow$ faster).
- **Damping ratio:** for a complex pair, angle ϕ from negative real axis gives $\zeta = \cos \phi$.
- **Natural frequency:** radius to origin
 $\omega_n = \sqrt{\alpha^2 + \omega^2}$.
- Quick links (2nd order): $T_p = \pi/\omega_d$,
 $\%OS = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$, $T_s \approx 4/(\zeta\omega_n)$.

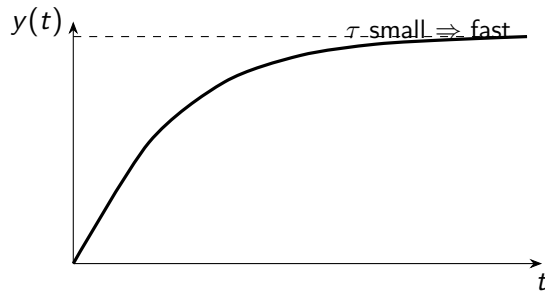


Example A: one real pole (first order)

$$T(s) = \frac{K}{\tau s + 1} \Rightarrow y(t) = K(1 - e^{-t/\tau})u(t).$$

Pole at $s = -1/\tau$:

- Larger τ (pole nearer $j\omega$) $\Rightarrow$ slower rise.
- Step response is monotone; no overshoot without zeros.



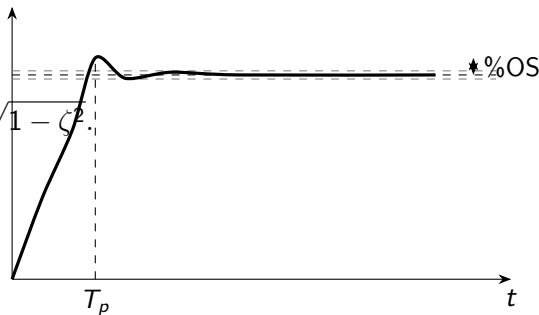
Example B: complex pair (second order, unit step)

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}.$$

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta), \quad \omega_d = \omega_n \sqrt{1-\zeta^2}.$$

Links:

$$T_p = \frac{\pi}{\omega_d}, \quad \%OS = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}, \quad T_s \approx \frac{4}{\zeta\omega_n}.$$

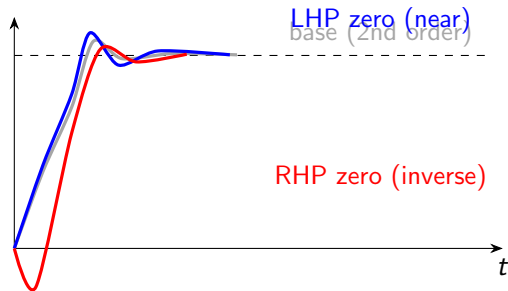


Zeros shape the transient (intuitive cases)

Case 1: zero on LHS (minimum phase)

$$T(s) = \frac{(s + a)\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad a > 0.$$

- If a is *near* dominant poles ($a \not\gg \zeta\omega_n$): more peaking/overshoot.
- If $a \rightarrow \infty$: zero “far left” $\Rightarrow$ negligible effect (looks 2nd order).



Case 2: zero on RHS (non-minimum phase)

$$T(s) = \frac{(s - a) \dots}{(\text{stable denom})}, \quad a > 0.$$

- *Inverse response*: output initially moves the wrong way.

Example C: read the s-plane, sketch the response

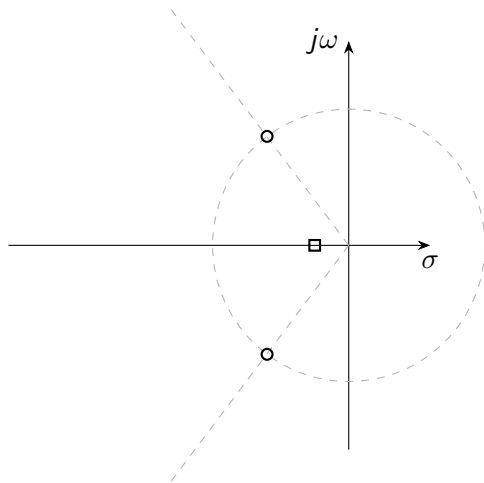
Given

$$p_{1,2} = -1.2 \pm j1.6, \quad \text{zero at } z = -0.5.$$

Step 1. $\omega_n = \sqrt{1.2^2 + 1.6^2} \approx 2.0$,
 $\zeta = 1.2/2.0 = 0.6$.

Step 2. Predict 2nd-order metrics: $T_p = \pi/\omega_d$,
 $\omega_d = 2 \cdot \sqrt{1 - 0.6^2} \approx 1.6 \Rightarrow T_p \approx 1.96 \text{ s}$;
 $\%OS \approx 9.5\%$, $T_s \approx 4/(\zeta\omega_n) \approx 3.3 \text{ s}$.

Step 3. Zero at -0.5 near pair $\Rightarrow$ expect a bit more peaking than pure 2nd order.



Dominant poles: when can I ignore others?

- If a real pole lies $\gtrsim 10\times$ farther left than the dominant pair's real part, its effect on step transients is usually small.
- Rule of thumb: keep DC gain (steady-state) consistent when neglecting fast poles/zeros.
- Always validate with a quick simulation or measured response.

Common patterns to recognize

- **Real poles only** $\Rightarrow$ no oscillation; sum of decays.
- **Complex pair** $\Rightarrow$ ringing; shape from ζ, ω_n .
- **LHP zero near poles** $\Rightarrow$ more peaking / faster initial rise.
- **RHP zero** $\Rightarrow$ inverse response (initial undershoot).
- **Pole-zero near cancellation** $\Rightarrow$ fragile; tiny mismatch can reintroduce dynamics—avoid relying on exact cancellation.

Poll: quick checks

Q1 Which pole set yields the fastest decay? (A) $-1 \pm j2$, (B) $-3 \pm j2$, (C) $-0.5 \pm j4$.

Q2 A system has a RHP zero at $s = +0.8$. What qualitative feature do you expect in a step response?

Q3 For poles at $-2 \pm j2$, estimate ζ and ω_n .

Q4 If $\zeta = 0.6$ and $\omega_n = 5$, estimate T_p and T_s .

Q5 You add a fast real pole at $s = -50$ to a system with dominant $-2 \pm j3$. Does T_s change much?

A1 (B) $-3 \pm j2$ (more negative real part $\Rightarrow$ fastest decay).

A2 Initial **inverse response** (undershoot) before rising toward the final value.

A3 $\omega_n = \sqrt{2^2 + 2^2} = \sqrt{8} \approx 2.83$; $\zeta = 2/\omega_n \approx 0.707$.

A4 $\omega_d = \omega_n \sqrt{1 - \zeta^2} = 5\sqrt{1 - 0.36} = 4$; $T_p = \pi/\omega_d \approx 0.785$ s; $T_s \approx 4/(\zeta\omega_n) = 4/3 \approx 1.33$ s.

A5 Hardly—fast non-dominant pole has minimal effect on dominant-mode settling (still check DC gain).

Takeaways

- $T_p, \%OS \leftrightarrow \zeta; T_s \leftrightarrow \zeta\omega_n$.
- Pole angle ϕ sets $\zeta = \cos \phi$; radius sets ω_n .
- Extra zeros/poles may perturb these relations—check dominance before reducing to 2nd order.

Poll: quick checks

Q1 For $\zeta = 0.5$, estimate %OS.

Q2 If $T_s \approx 1.6$ s and $\zeta = 0.5$, estimate ω_n .

Q3 A first peak occurs at $T_p = 0.7$ s; estimate $\omega_n \sqrt{1 - \zeta^2}$.

Q4 True/False: The impulse response is the time derivative of the *input* step (not the output).

Q5 Increasing ω_n with fixed ζ primarily affects which metric?

A1 $\%OS = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100e^{-0.5\pi/0.866} \approx 16\%$. :contentReference[oaicite:20]index=20

A2 $T_s \approx 4/(\zeta\omega_n) \Rightarrow \omega_n \approx 4/(0.5 \times 1.6) = 5 \text{ rad/s}$.

A3 $\omega_n\sqrt{1-\zeta^2} = \pi/T_p \approx \pi/0.7 \approx 4.49 \text{ rad/s}$.

A4 **False**. The impulse *output* is the time derivative of the *step output* (same system).

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A5 **Settling time** (faster decay), while $\%OS$ is mainly a function of ζ .

Effects of a Third Pole and a Zero: Why this matters

- Many real systems are *not* exactly second order, but have a dominant complex pair. Their step response can often be *approximated* well by a 2nd-order template.
- A **third (real) pole** far to the left (fast) usually has little time-domain effect. A **finite zero** near the dominant pair can materially reshape the transient (overshoot, undershoot).

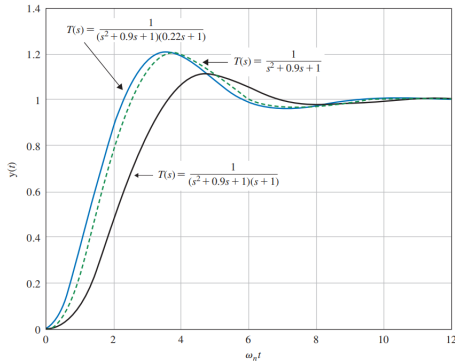


FIGURE 5.12
Comparison of two third-order systems with a second-order system (dashed line) illustrating the concept of dominant poles when $|1/\gamma| \geq 10\zeta\omega_n$.

Third-order template and the dominant-pole criterion

Normalized 3rd-order form (unity feedback):

$$T(s) = \frac{1}{(s^2 + 2\zeta s + 1)(\gamma s + 1)} \quad (\text{with } \omega_n = 1 \text{ after normalization}).$$

A practical approximation is valid when the extra real pole is sufficiently *fast*:

$$\boxed{\frac{\gamma}{\zeta\omega_n} \geq 10}$$

i.e., the real part of the third pole is at least ten times more negative than that of the dominant pair. Then the 2nd-order metrics (P.O., T_s , T_r) predict the 3rd-order step response well.

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Worked micro-example: zero + third pole

Given:

$$T(s) = \frac{(s + a)\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(\tau s + 1)}.$$

Heuristics:

- ① If $\frac{\tau}{1/(\zeta\omega_n)} \leq 0.1$ (i.e., $\frac{\gamma}{\zeta\omega_n} \geq 10$ with $\gamma = \tau$), neglect the real pole for transients.
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- ② If $\frac{\zeta\omega_n}{a} \leq 0.1$, the zero is far left and its effect is small; otherwise expect higher overshoot or initial undershoot. :contentReference[oaicite:10]index=10
- ③ Always check DC gain when neglecting elements to keep e_{ss} predictions consistent.
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Textbook numerical illustration

For $\omega_n = 1$, $\zeta = 0.45$:

- Case $\gamma = 1.0$: extra pole at $s = -1.0$; P.O. $\approx 10.9\%$, $T_s \approx 8.84$ s, $T_r \approx 2.161$ s.
- Case $\gamma = 0.22$: extra pole at $s = -4.5$; dominant pair governs, P.O. $\approx 20.0\%$, $T_s \approx 8.56$ s, $T_r \approx 1.6$ s; 2nd-order approximation is accurate when $\gamma/(\zeta\omega_n) \geq 10$.

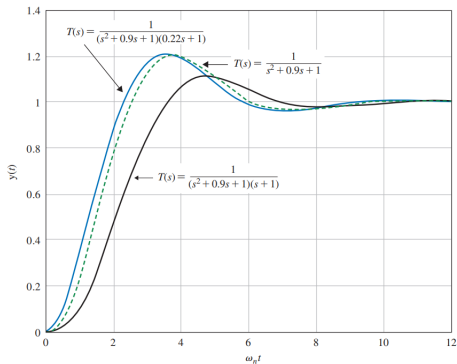


FIGURE 5.12
Comparison of two
third-order systems
with a second-order
system (dashed
line) illustrating the
concept of dominant
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 $|\gamma| \geq 10\zeta\omega_n$.

Reduction to an effective second order (step-by-step)

Start: $T(s) = \frac{N(s)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(\gamma s + 1)}$ with $\frac{\gamma}{\zeta\omega_n} \geq 10$.

- (1) Neglect $(\gamma s + 1)$ in the *transient* path; retain DC gain if needed.
- (2) If a zero exists at $s = -a$ and $\zeta\omega_n/a \ll 1$, ignore its transient effect.
- (3) Use 2nd-order formulas for P.O., T_p , T_s , T_r ; validate by simulation/experiment.

Poll: quick checks

- Q1 The third pole can be neglected for transients when $\gamma/(\zeta\omega_n)$ is approximately $\geq$ what value?
- Q2 A zero at $s = -a$ with $\zeta\omega_n/a = 2$ is (near / far) and will (strongly / weakly) affect overshoot.
- Q3 True/False: A zero far left (large a) has negligible effect on the 2nd-order step response.
- Q4 When you drop a fast pole, what must you still verify regarding steady-state behavior?
- Q5 If two systems share the same dominant complex pair but one has an extra pole at $s = -5(\zeta\omega_n)$, which has the larger T_s ?

A1 $\gamma/(\zeta\omega_n) \geq 10$.

A2 **Near, strongly** (expect larger overshoot or even inverse response).

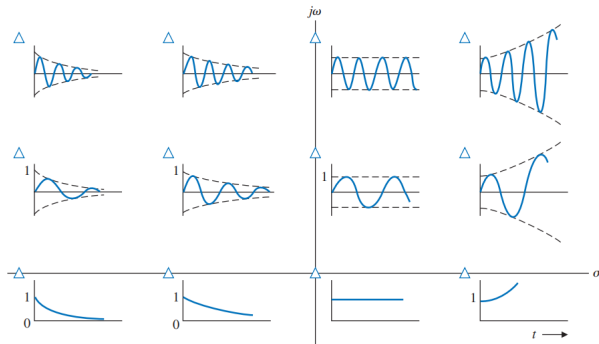
A3 **True**: as $a \rightarrow \infty$ the curve approaches the 2nd-order response.

A4 Check **DC gain**/type to keep e_{ss} predictions consistent.

A5 The one with the **same** dominant pair has essentially the **same** T_s (fast non-dominant pole has minor effect), provided $\gamma/(\zeta\omega_n) \geq 10$.

Big idea: poles $\leftrightarrow$ transient response

- Closed-loop transient response is determined by the **poles**
- For unity feedback, poles are the roots of $\Delta(s) = 0$ and they define $T(s) = Y(s)/R(s)$.
- Without repeated roots, a unit-step output decomposes by partial fractions (real and complex terms) and in time becomes a sum of decaying exponentials and damped sinusoids.



From poles to time response (step-by-step)

Partial fraction (no repeated roots): for step input and DC gain = 1,

$$Y(s) = \sum_{i=1}^M \frac{A_i}{s - \sigma_i} + \sum_{k=1}^N \frac{B_k s + C_k}{(s - \alpha_k)^2 + \omega_k^2}. \quad (\text{cf. Eq. (5.21)})$$

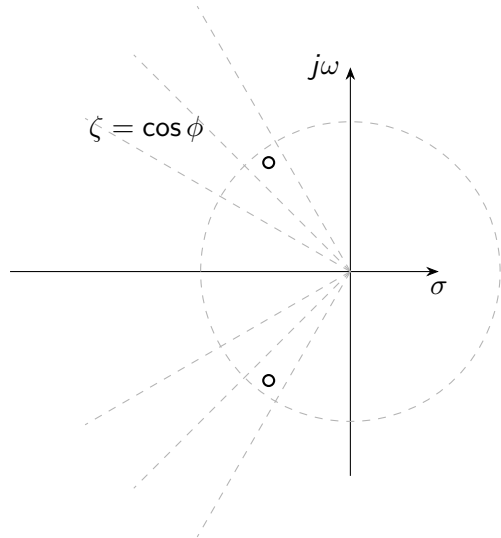
Inverse Laplace:

$$y(t) = \sum_{i=1}^M A_i e^{\sigma_i t} + \sum_{k=1}^N D_k e^{\alpha_k t} \sin(\omega_k t + \theta_k). \quad (\text{Eq. (5.22)})$$

- Real pole $s = \sigma_i$: pure decay $e^{\sigma_i t}$.
- Conjugate pair $s = \alpha_k \pm j\omega_k$: damped sinusoid $e^{\alpha_k t} \sin(\omega_k t + \theta_k)$.

s-plane geometry & what it tells us (tutorial)

- Distance to $j\omega$ -axis ($|\operatorname{Re}\{s\}|$) $\Rightarrow$ decay rate (speed).
- Angle ϕ from $-\sigma$ -axis sets damping ratio: $\zeta = \cos \phi$ (2nd-order).
- Radius to origin $\Rightarrow \omega_n$ (2nd-order); $\omega_d = \omega_n \sqrt{1 - \zeta^2}$.
- Use $(\%OS, T_p, T_s) \leftrightarrow (\zeta, \omega_n)$ links to sketch acceptable pole regions.



Estimating ζ from the waveform (count cycles)

For the 2nd-order step response, $\omega_d = \omega_n \sqrt{1 - \zeta^2}$ and the decay time constant is $\tau = 1/(\zeta \omega_n)$. Thus the number of cycles in one time constant is

$$\frac{\omega_d}{2\pi} \tau = \frac{\sqrt{1 - \zeta^2}}{2\pi\zeta}. \quad (\text{Eq. (5.19) context}) : \text{contentReference}[oaicite : 7]index = 7$$

Assuming the response is within 2% after $n = 4$ time constants gives the **visible cycles** estimate

$$\boxed{\text{cycles}_{\text{visible}} \approx \frac{2\sqrt{1 - \zeta^2}}{\pi\zeta}} \Rightarrow \zeta \text{ from a measured plot. (Eq. (5.20))}$$

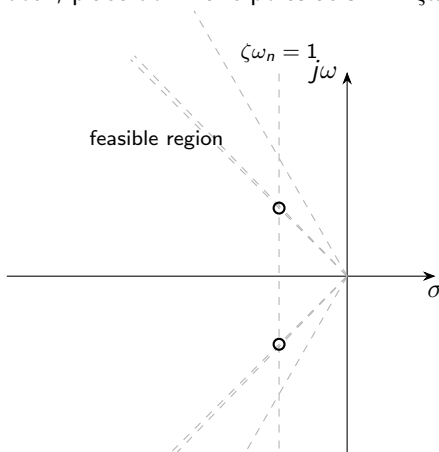
Alternative: use %OS $\Rightarrow \zeta$ via Eq. (5.16).

Spec $\rightarrow$ pole region (worked micro example)

Specs: $\%OS \leq 5\%$ and $T_s \leq 4 \text{ s } (\pm 2\%)$. Then

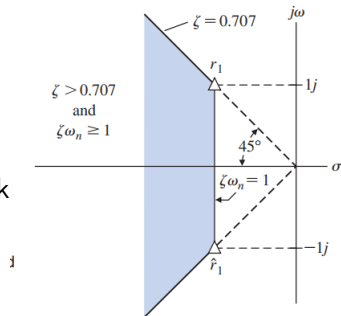
$$\zeta \geq 0.69 \quad \text{and} \quad \zeta\omega_n \geq 1.$$

Choose a point satisfying both; place dominant poles at $s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$. (Matches textbook

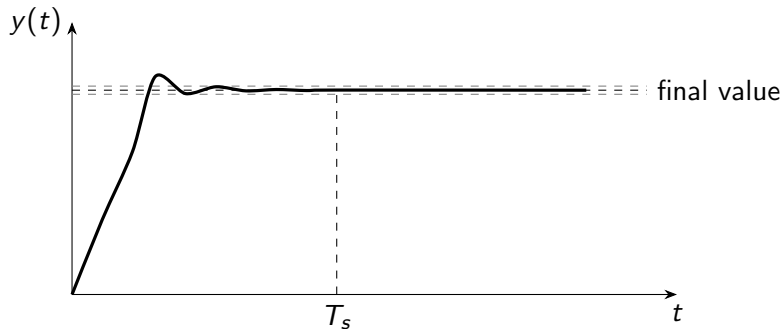


Reading an s-plane plot $\rightarrow$ time response (skills drill)

- **Given poles:** estimate $\zeta = \cos \phi$ and $\omega_n = \sqrt{\alpha^2 + \omega_d^2}$.
- **Predict:** $\%OS(\zeta)$, $T_p = \pi/\omega_d$, $T_s \approx 4/(\zeta\omega_n)$.
- **Refine:** if additional poles/zeros exist, check dominance and zero placement (see §5.4).



Practice: count cycles to estimate ζ (visual)



Count visible oscillations until T_s (about 4 time constants) and use

$$\text{cycles}_{\text{visible}} \approx \frac{2\sqrt{1-\zeta^2}}{\pi\zeta} \Rightarrow \zeta \text{ estimate.}$$

Poll: quick checks

Q1 Which s-plane feature most directly controls decay speed: (A) angle, (B) distance to $j\omega$ axis, (C) radius?

Q2 For a complex pair at $s = -3 \pm j4$, estimate ζ and ω_n .

Q3 If the step response shows ≈ 1.3 visible cycles up to T_s , estimate ζ (use Eq. (5.20)).

Q4 True/False: For closed-loop systems, the transient response depends only on zeros.

Q5 Given $T_p = \pi/\omega_d$ and $T_s \simeq 4/(\zeta\omega_n)$, which move in the s-plane reduces T_s without changing %OS?

A1 (B) Distance to the $j\omega$ -axis (larger $|\operatorname{Re}\{s\}| \Rightarrow$ faster decay).

A2 $\omega_n = \sqrt{3^2 + 4^2} = 5$; $\zeta = 3/5 = 0.6$.

A3 Solve $1.3 \approx \frac{2\sqrt{1-\zeta^2}}{\pi\zeta} \Rightarrow \zeta \approx 0.45$ (approx). :contentReference[oaicite:17]index=17

A4 False — poles (roots of $\Delta(s) = 0$) govern transients; zeros can shape but do not alone determine stability/transient. :contentReference[oaicite:18]index=18

A5 Increase ω_n at fixed ζ (move poles radially outward along a constant- ζ ray). $T_s \downarrow$, %OS (set by ζ) unchanged.

The Steady-State Error of Feedback Control Systems: Goal & tools

- **Goal:** quantify tracking accuracy for standard inputs (step/ramp/parabola).
- **Tool:** Final Value Theorem (assumes closed-loop stability):

$$e_{ss} = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s).$$

- **Idea:** define *error constants* (K_p, K_v, K_a) and *type* N (number of pure integrators in $G_c(s)G(s)$) to predict e_{ss} .

(See section intro and definitions.)

Unity feedback: setup and definitions (type and constants)

Unity negative feedback, reference R , controller G_c , plant G :

$$E(s) = \frac{R(s)}{1 + G_c(s)G(s)}, \quad e_{ss} = \lim_{s \rightarrow 0} \frac{s R(s)}{1 + G_c(s)G(s)}.$$

Define loop $L(s) = G_c(s)G(s)$. With N = number of integrators in $L(s)$:

$$K_p = \lim_{s \rightarrow 0} L(s), \quad (\text{position})$$

$$K_v = \lim_{s \rightarrow 0} s L(s), \quad (\text{velocity})$$

$$K_a = \lim_{s \rightarrow 0} s^2 L(s). \quad (\text{acceleration})$$

Standard inputs $\Rightarrow$ steady-state error (unity feedback)

Step $r(t) = A u(t)$, $R(s) = A/s$:

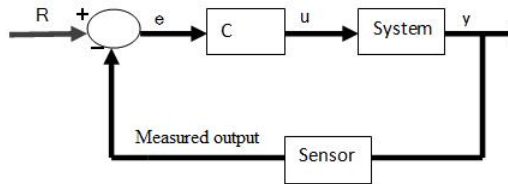
$$e_{ss} = \lim_{s \rightarrow 0} \frac{A}{1 + L(s)} = \frac{A}{1 + K_p}.$$

Ramp $r(t) = At u(t)$, $R(s) = A/s^2$:

$$e_{ss} = \lim_{s \rightarrow 0} \frac{A}{s(1 + L(s))} = \frac{A}{K_v} \quad (N \geq 1).$$

Parabola $r(t) = \frac{A}{2} t^2 u(t)$, $R(s) = A/s^3$:

$$e_{ss} = \lim_{s \rightarrow 0} \frac{A}{s^2(1 + L(s))} = \frac{A}{K_a} \quad (N \geq 2).$$



What system *type* tells you (table view)

Type N (integrators in L)	Step A	Ramp At	Parabola $\frac{A}{2}t^2$
0	$\frac{A}{1 + K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2	0	0	$\frac{A}{K_a}$

(Summary consistent with Table 5.2.) :contentReference[oaicite:11]index=11

Ramp & parabolic inputs: constants and limits

For a **type-1** loop ($N=1$),

$$K_v = \lim_{s \rightarrow 0} s L(s), \quad e_{ss}^{\text{ramp}} = \frac{A}{K_v}.$$

For a **type-2** loop ($N=2$),

$$K_a = \lim_{s \rightarrow 0} s^2 L(s), \quad e_{ss}^{\text{parabola}} = \frac{A}{K_a}.$$

If $N \geq 3$, then $e_{ss} = 0$ even for parabolic input.

Non-unity feedback: reduce to unity via $Z(s)$

For the non-unity loop with sensor $H(s)$, we can form an **equivalent unity** loop with

$$Z(s) = \frac{G_c(s)G(s)}{1 - (1 - H(s))G_c(s)G(s)}.$$

Then the error constants are

$$K_p = \lim_{s \rightarrow 0} Z(s), \quad K_v = \lim_{s \rightarrow 0} s Z(s), \quad K_a = \lim_{s \rightarrow 0} s^2 Z(s),$$

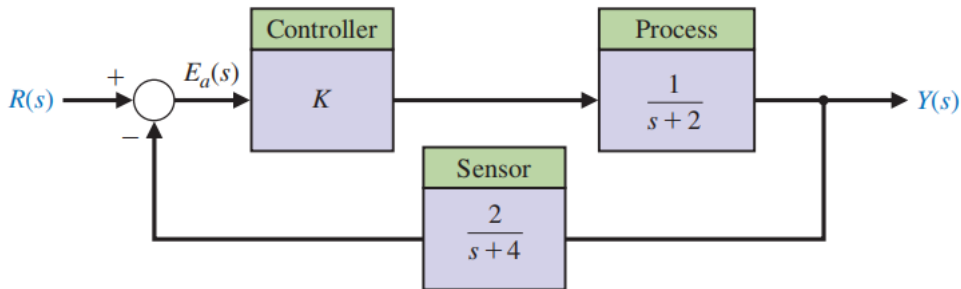
and reduce to the unity-feedback formulas when $H(s) = 1$. (See derivation and Eq. set for $Z(s)$.)

Example (textbook style): choose gain to meet step e_{ss}

Closed-loop $T(s) = \frac{G_c G}{1 + G_c G H}$, step error

$$e_{ss} = 1 - T(0).$$

For the given G_c , G , H in the text example, a zero step error requires $T(0) = 1 \Rightarrow$ an algebraic condition on the gains; e.g., $K_0 + 2K_4 = 8$ (illustrative), yielding a feasible K .



Mini-example: PI makes step error zero (type $\uparrow$)

Let $G_c(s) = K_1 + \frac{K_2}{s}$, $G(s)$ proper. Then $L(s) = G_c G$ has one integrator $\Rightarrow$ type-1:

$$e_{ss}^{\text{step}} = 0, \quad e_{ss}^{\text{ramp}} = \frac{A}{K_v}, \quad K_v = \lim_{s \rightarrow 0} s G_c G = K_2 G(0).$$

Trade-off: larger K_2 (better ramp tracking) can alter damping—verify transients (§5.3–5.4 links). (Robot steering example context.)

Design checklist (step-by-step)

- 1 Decide **required type** (0/1/2) from input class you must track.
- 2 Compute K_p, K_v, K_a targets from **allowable** e_{ss} for step/ramp/parabola.
- 3 Synthesize $G_c(s)$ (e.g., PI to raise type; lead/lag to shape DC gain).
- 4 Validate: recompute e_{ss} , confirm stability (needed for final-value theorem).

(Unity/non-unity handling; stability caveat.) :contentReference[oaicite:18]index=18

Poll: quick checks

Q1 A type-0 system with $K_p = 9$ receives a step of amplitude A . What is e_{ss} ?

Q2 For type-1, write the formula for the ramp error and the constant it uses.

Q3 For type-2, what is the parabolic error and the corresponding constant?

Q4 In non-unity feedback with sensor $H(s)$, which loop do you use to define K_p, K_v, K_a ?

Q5 A PI controller changes the system type from what to what (most commonly), and what happens to the step error?

A1 $e_{ss} = \frac{A}{1 + K_p} = \frac{A}{10}$. (Step error form.) :contentReference[oaicite:19]index=19

A2 $e_{ss} = \frac{A}{K_v}$ with $K_v = \lim_{s \rightarrow 0} s L(s)$. (Velocity constant.) :contentReference[oaicite:20]index=20

A3 $e_{ss} = \frac{A}{K_a}$ with $K_a = \lim_{s \rightarrow 0} s^2 L(s)$. (Acceleration constant.) :contentReference[oaicite:21]index=21

A4 Use the **equivalent unity** loop with $Z(s)$; $K_p = \lim_{s \rightarrow 0} Z(s)$, etc. (Non-unity reduction.) :contentReference[oaicite:22]index=22

A5 Typically type-0 $\rightarrow$ type-1; step error goes to **zero**. (Type increase via integrator.) :contentReference[oaicite:23]index=23

Why performance indices?

- We want a **single number** that summarizes time-domain quality (to tune parameters or compare designs).
- Choose an index that is **nonnegative** and emphasizes what you care about; then **minimize** it.

(Intro + motivation.) :contentReference[oaicite:0]index=0

The four classics (definitions)

For error $e(t) = r(t) - y(t)$ over $t \in [0, T]$ (often take $T = T_s$):

$$\mathbf{ISE} : \int_0^T e^2(t) dt, \quad \mathbf{IAE} : \int_0^T |e(t)| dt$$

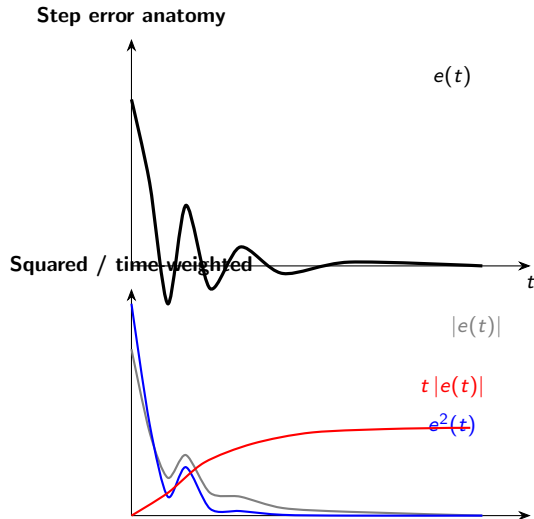
$$\mathbf{ITAE} : \int_0^T t |e(t)| dt, \quad \mathbf{ITSE} : \int_0^T t e^2(t) dt$$

ISE is algebraically convenient; ITAE de-emphasizes large *initial* errors and highlights *late* errors, often giving the most selective minimum. (Defs + comments.)

What each index *likes* (weighting intuition)

- **ISE**: squares spikes; penalizes large overshoot strongly; tolerant to small late drift.
- **IAE**: robust to spikes; trades off width vs. height of error lobe.
- **ITSE**: like ISE but shifts attention later in time.
- **ITAE**: explicitly weights *late* error; tends to reduce settling tails.

(General form and rationale: pick the weighting to match specs.) :contentReference[oaicite:3]index=3



Design workflow (step-by-step)

Step 1 Choose T (often T_s from your spec).

Step 2 Select an index aligned with your priority (overshoot vs. tail).

Step 3 Express $e(t)$ from your loop $L(s)$ (unity or equivalent-unity form).

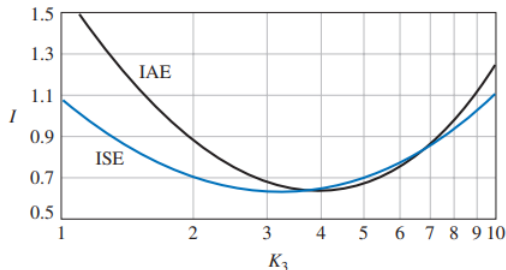
Step 4 Compute the integral; search parameters for the minimum.

Step 5 Validate: check time specs (T_p , %OS, T_s) and steady-state error.

Illustration: different indices $\Rightarrow$ different optima

A telescope pointing loop is tuned against ISE and IAE; the minimizing settings *differ*. At the ISE minimum the loop is less heavily damped (e.g., $\zeta \approx 0.5$); at the IAE minimum it is more damped (e.g., $\zeta \approx 0.665$). This shows that the “best” design depends on the chosen index.

FIGURE 5.25
The performance indices of the telescope control system versus K_3 .



General form and variants

A performance index can be written in general as a **weighted integral** of error and related signals; classical choices yield ISE/IAE/ITSE/ITAE when weights are 1, 1, t , t with squares/absolutes as shown. (General performance-integral idea.)

ITAE optimal prototypes (step input)

For zero steady-state error to a step and *no finite zeros*, closed-loop prototypes

$$T(s) = \frac{\omega_n^n}{s^n + b_{n-1}\omega_n s^{n-1} + \cdots + b_1\omega_n^{n-1}s + \omega_n^n}$$

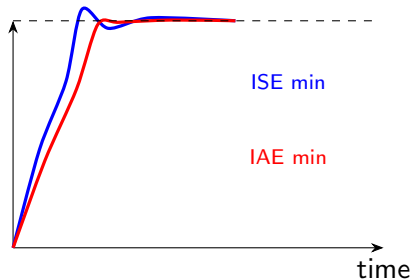
with coefficients b_k chosen to **minimize ITAE** produce fast, low-tail responses (normalized in $\omega_n t$).
(Form + “no zeros”, coefficients in Table 5.3, responses in Fig. 5.26.)

n	normalized denominator (examples)
2	$s^2 + 1.4\omega_n s + \omega_n^2$
3	$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$
4	$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$

(Representative rows paraphrased from Table 5.3.)

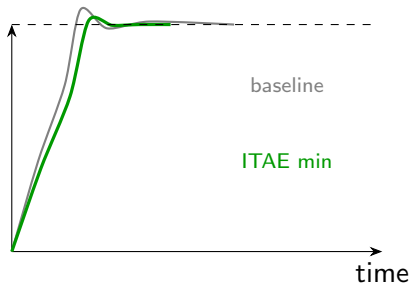
Visualizing the differences (TikZ mini-gallery)

(a) ISE vs. IAE tuning



(Selectivity of indices and late-error emphasis.)

(b) ITAE emphasis



ITAE optimal prototypes (ramp input)

For tracking ramps with zero e_{ss} , another normalized family minimizes ITAE for a ramp input:

$$T(s) = \frac{b_n \omega_n^n}{s^n + b_{n-1} \omega_n s^{n-1} + \dots + b_0 \omega_n^n}$$

(see Table 5.4 in the text for coefficients; requires process with ≥ 2 pure integrations).

(Statement + requirement; see section text.)

Step-by-step micro-tuning example (recipe)

Goal: minimize ITAE for a 2nd–3rd order loop with unity feedback.

1. Choose $T = T_s$ from your spec; get $e(t)$ from simulation/analytics.
2. Evaluate $\text{ITAE} = \int_0^T t |e(t)| dt$ while sweeping the gain(s).
3. At the minimum, read ζ, ω_n , then check $T_p, \%OS, T_s$.
4. Optionally compare to the ITAE prototype denominator for your n to guide retuning.

Poll: quick checks

Q1 Which index *most* emphasizes late-time errors? (ISE, IAE, ITSE, ITAE)

Q2 Why is ITAE often called “selective”?

Q3 If your design has small overshoot but a long tail, which index will push you to shorten the tail?

Q4 For a step-tracking closed loop with no finite zeros, what's special about the ITAE prototype denominators?

Q5 True/False: Choosing a different index can change the optimal damping ratio for the same plant.

A1 ITAE (the t factor weights late error).

A2 Its minimum is easy to discern as parameters vary (good discrimination).

A3 ITAE (and ITSE) shift emphasis later; ITAE is strongest for tails.

A4 They have n poles and *no zeros*, with coefficients chosen to minimize ITAE for a step. A5 **True** — different indices (ISE vs. IAE vs. ITAE) can yield different optimal settings.

- EXERCISES E5.10, E5.15
- PROBLEMS P5.2, P5.13,

Submit homework to **homeworkscut2026@126.com**; **due Oct. 8, 2026.**